# *Application of the credibility principle in reinsurance pricing*

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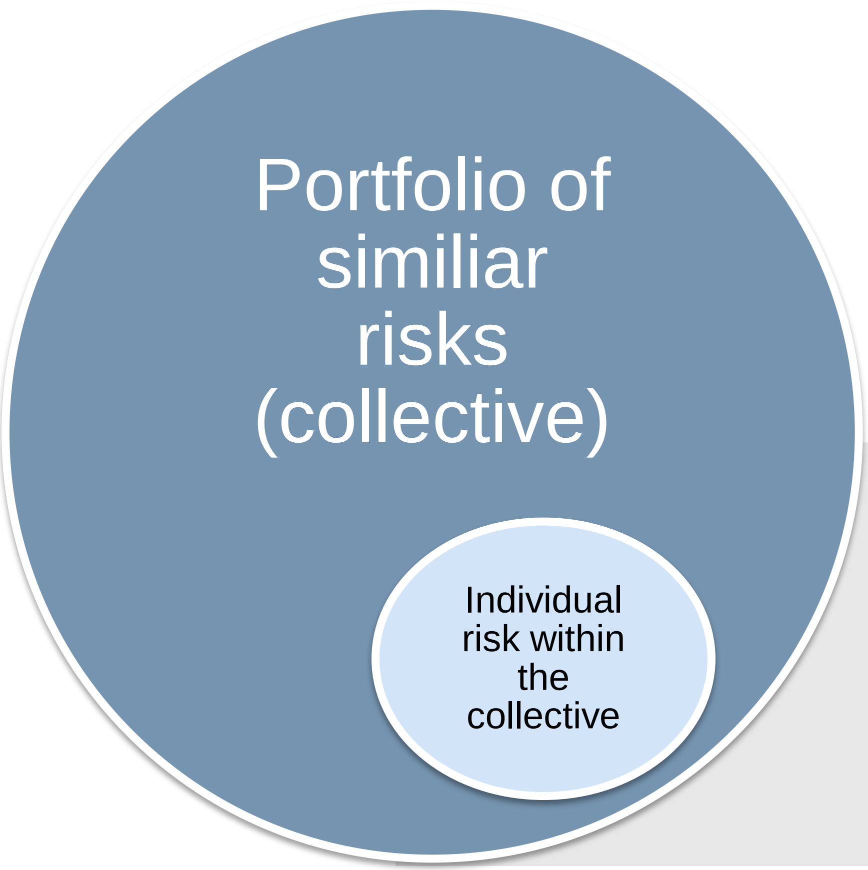
# *Agenda*

1. Introduction into credibility theory
2. Some maths
3. Credibility for reinsurance pricing
4. Application – method used for MTPL
5. Vision

# *Introduction*

# Introduction




Initial situation:

- ➢Comprehensive information available for the collective (e.g. solid loss history or more)
- ➢Limited data history available for individual risk

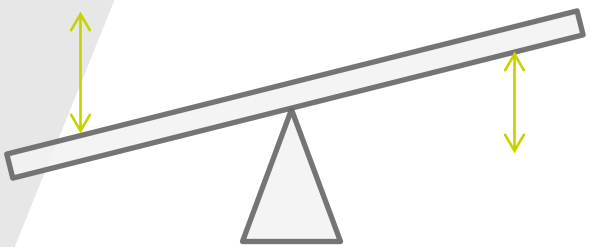
GOAL:

Make use of all (relevant) available information in order to get best estimate for the individual premium

# *Introduction*
# *Collective vs individual information*

| Individual information | Collective information |
|---|---|
| Data stems from individual risk | Different charateristics than individual risk |
| Contains significant random element | Statistical significance |

$$Z \cdot \bar{X} + (1 - Z) \cdot m$$



Credibility premium

# Introduction
# History of credibility theory

- Limited Fluctuation CT
  - Based on central limit theorem
  - Originally a "full or zero-credibility" method
  - Parameters in partial model introduced later calibrated according to actuarial judgement

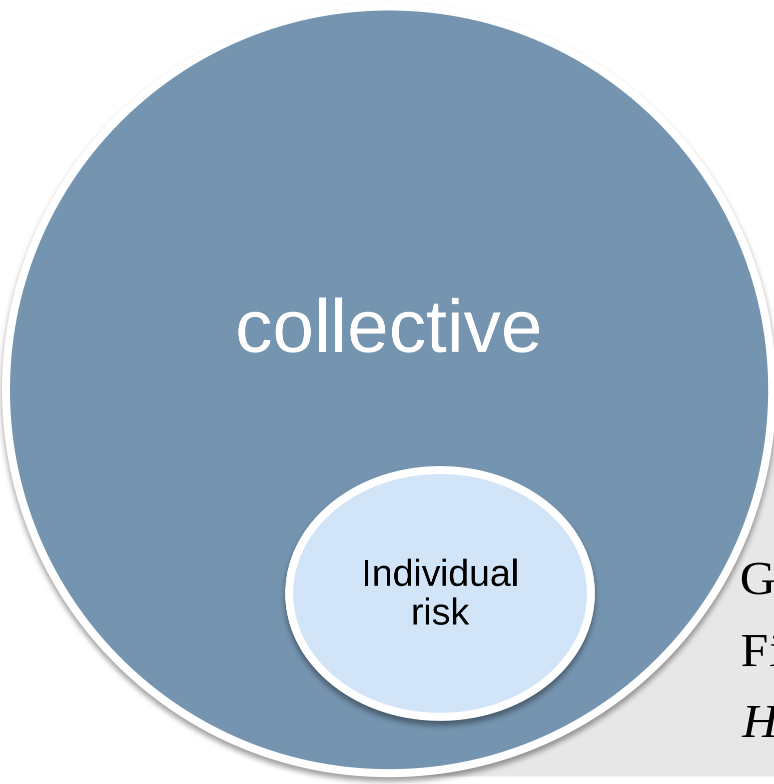
**Stability-oriented approach**

- Greatest Accuracy CT
  - Heavily based on Bayesian statistics
  - "Best premium to charge"-approach
  - Results in stable and responsive estimator

**Precision-oriented approach**

## *Some maths*

# *Mathematical Formulation*




$$\mathrm{F} = \{F_{\vartheta} \mid \vartheta \in \Theta\}$$

Family of distributions indexed by risk profile ϑ

$$H[\Theta] = E[X \mid \Theta]$$

Individual premium

$$\mu = E[X] = E_{\Theta}\left(E[X \mid \Theta]\right)$$

Collective premium

GOAL:

Given observations $x_1, \ldots, x_n$ for an individual risk $F_{\vartheta}$

Find a good estimator for the individual premium

$$H(\vartheta) = E[X_{n+1} \mid \vartheta]$$

# Main results – Bayesian estimator

$$\pi(\vartheta)$$

A priori density function of risk profile

$$f(x_1, \ldots, x_n \mid \vartheta)$$

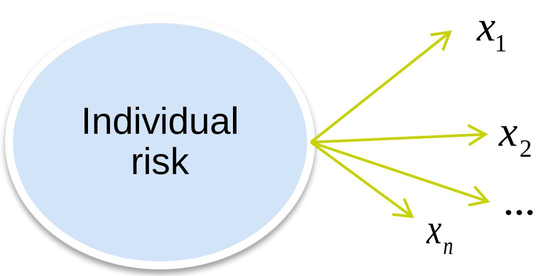
Conditional density function of losses

$$f(x_1, \ldots, x_n, \vartheta)$$

Joint density function

$$H(\vartheta) = \int x \cdot f(x \mid \vartheta) dx$$

Individual Premium




$$\pi(\vartheta \mid \mathbf{x})$$

A posteriori pdf of risk profile

$$P(B) = \sum_i P(B \mid A_i) \cdot P(A_i)$$

$$= \mathbf{x}) = \int H($$

$$P(A, B) = P(A \mid B) \cdot P(B)$$

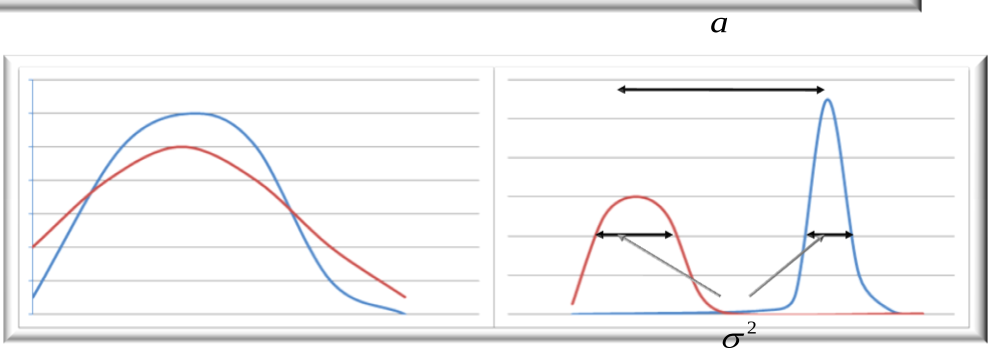
# *Main results – Bühlmann model*

collective observations

$$\hat{\mu}(\Theta) = (1 - Z_n)\mu + Z_n \overline{X}_n$$

$$Z_n = \frac{n}{n+k}$$

$$k = \frac{\sigma^2}{a}$$




more weight assigned to collective information

more weight assigned to individual information

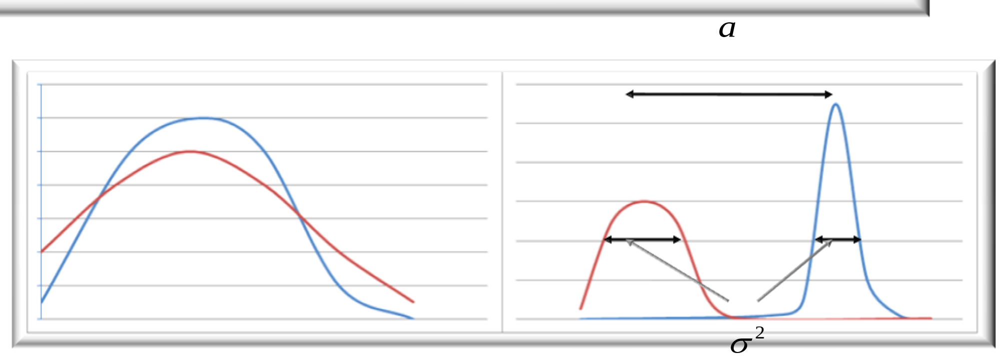
# *Main results – Bühlmann-Straub model*

collective observations

$$\hat{\mu}^{w}(\Theta) = \left(1 - Z_n^{w}\right)\mu + Z_n^{w}\,\overline{X}_n^{w}$$

$$Z_n^{w} = \frac{w}{w+k}$$

$$k = \frac{\sigma^2}{a}$$




more weight assigned to collective information

more weight assigned to individual information

# *Credibility for reinsurance pricing*

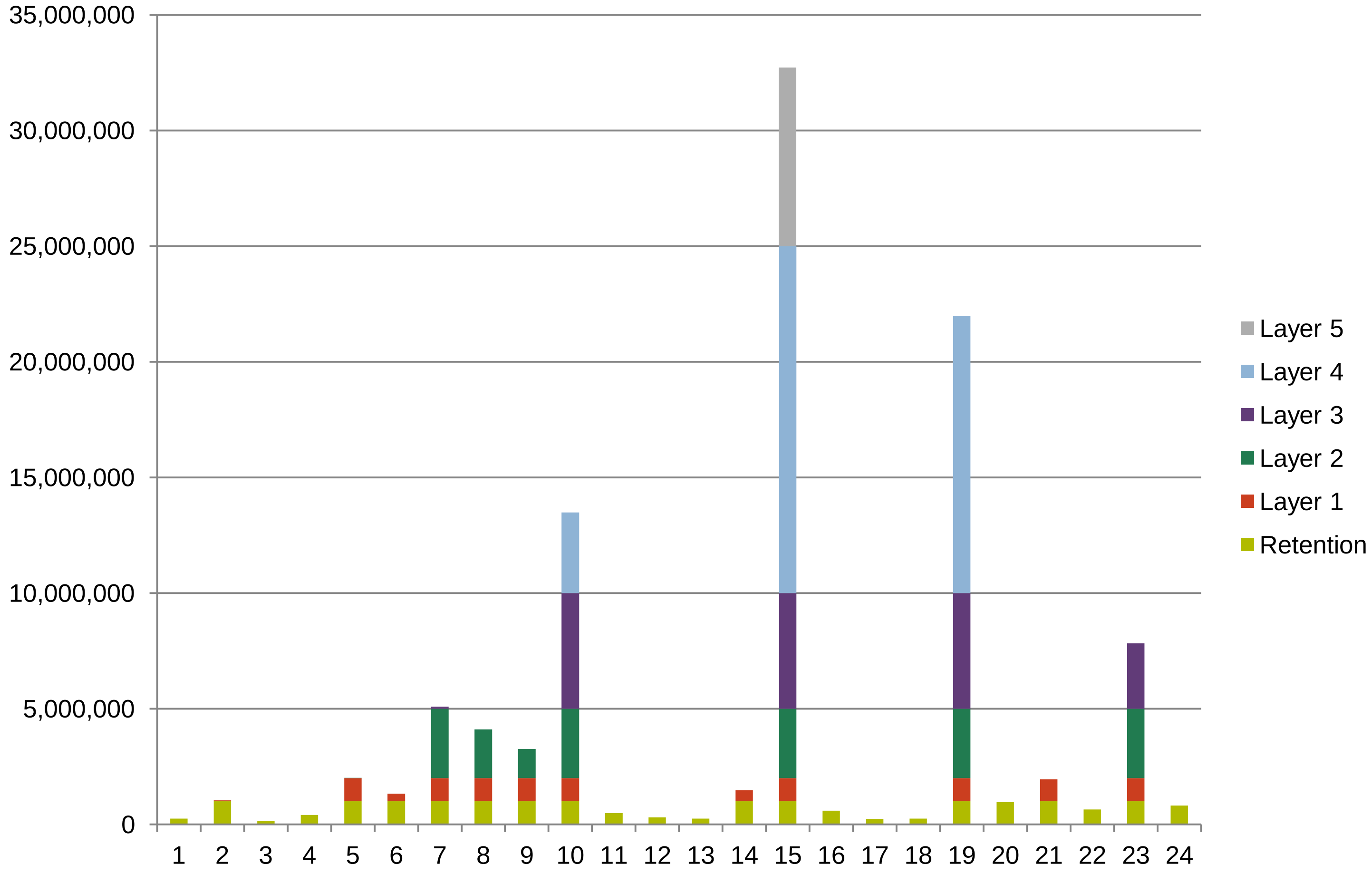
# *General comments*

What problems do we face in reinsurance pricing?

- Pricing XL business for motor
- Usually data are only given back for the last 10 years
- Need to project losses to ultimate, where development can take much longer than 10 years
- Data are only available  excess a threshold
- Hence scarce data, which may be insufficient to price a client based on experience
- We want to make use of all available data in market and weight a client against the market

naturally a application field of credibility

# General comments



# *Challenges*

Challenges:

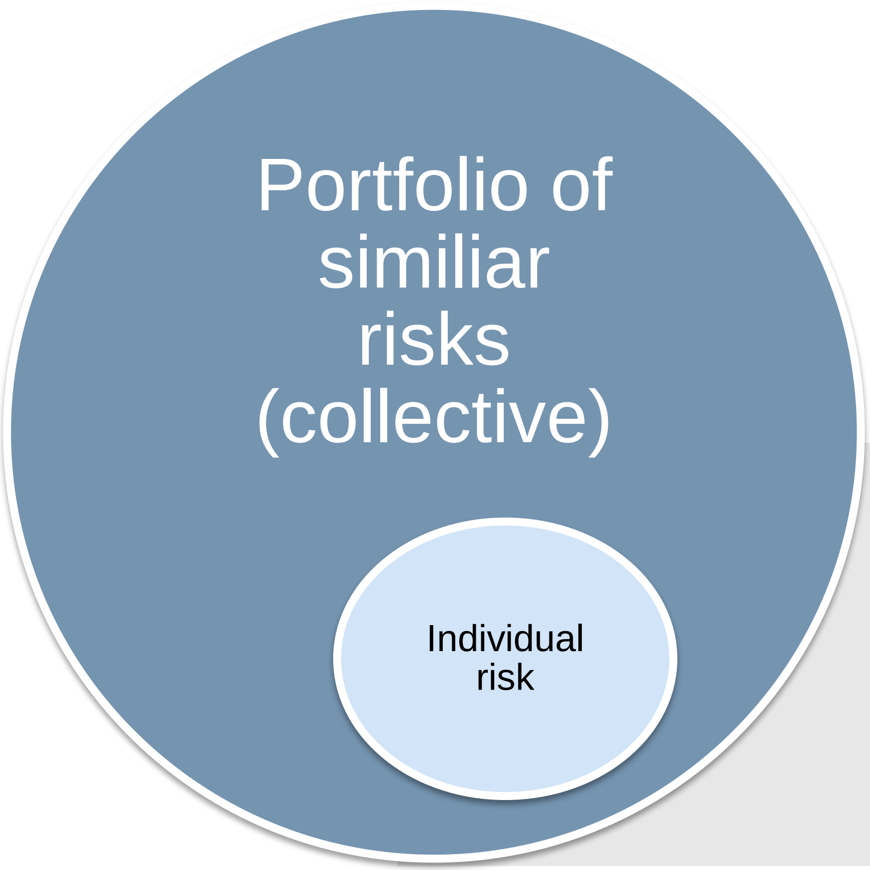
- Choice of appropriate portfolio
- Pure niche portfolios still require individual treatment
- Pricing of a layer 3 m xs 2 m is different than pricing ill xs 25 m
- Credibility weight needs to be calculated in dependency of claims size
- Credibility applied to frequency / severity or rate?
- Parameter uncertainty?
- Which is the appropriate exposure measure?

# *Requirements for credibility approach*

- Produce reasonable results i.e. increase precision
- Ensure stability and responsiveness
- One model for all layers
- Easy to explain
- Ensure consistent approach within one market
- Application still allows for underwriting judgement

*"Any credibility procedure requires the actuary to exercise informed judgment, using relevant information. The use of credibility procedures is not always a precise mathematical process" (Actuarial Standards board)*

# *Credibility for reinsurance XoL pricing*




Initial situation:

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- Limited data history available for individual risk

GOAL:

Make use of all (relevant) available information in order to get best estimate for the individual premium

# *Credibility for reinsurance XoL pricing*


Initial situation:

- ➢Net Market rate available (=Collective information)
- ➢Limited loss history available for individual portfolio

GOAL:

Make use of both information in order to get good estimate for the individual net rate

# *Application – Method used for MTPL*

# Theoretical framework – Compound Gamma-Poisson

$$f_N(n|\theta) = \frac{\theta^n e^{-\theta}}{n!}$$

$$f_\Theta(\theta) = \frac{b^a \theta^{a-1} e^{-b\theta}}{\Gamma(a)}$$

The unconditional distribution of *N* is negative binomial with parameter(a, b /(1+b)).

$$=> \hat{\mu}(\Theta_l) = (1 - Z_l)\overline{N}^{w,Z} + Z_l \overline{N}_l^{w}$$

$$Z_l = \frac{w_l}{w_l + b}$$

# Estimation of the parameters-Compound Gamma model

Estimation of parameter b:

$$E[\Theta] = a/b = \frac{1}{m}\sum_{l=1}^{m}\lambda_l$$

$$\mathrm{Var}\left[\Theta\right] = a/b^2 = \frac{1}{m-1}\sum_{l=1}^{m}\left(\lambda_l - \overline{\lambda}\right)^2$$

$$=> b = \frac{\overline{\lambda}}{\frac{1}{m-1}\sum_{l=1}^{m}\left(\lambda_l - \overline{\lambda}\right)^2}$$

Process needs to be repeated for different thresholds

# *Estimation of the parameters-Compound Gamma model*

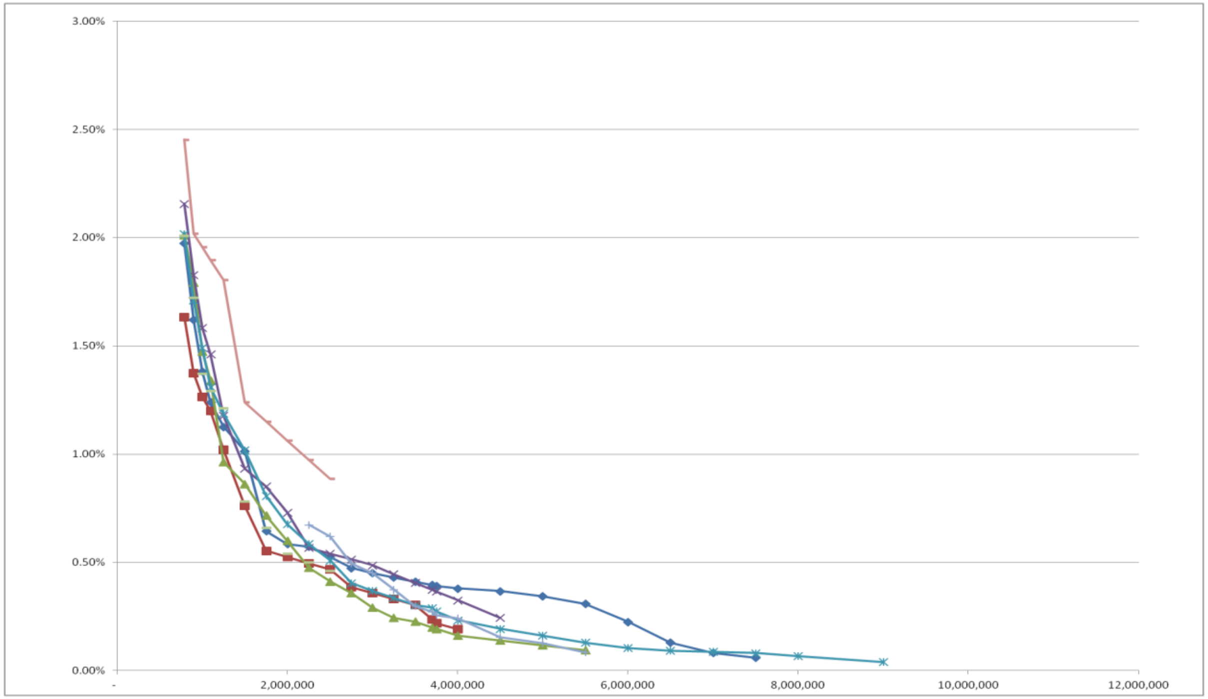
Estimation of parameter b(T):

$$\mathrm{E}[\Theta_T] = a(T)/b(T) = \frac{1}{m}\sum_{l=1}^{m}\lambda_l(T)$$

$$\mathrm{Var}\left[\Theta_T\right] = a(T)/b(T)^2 = \frac{1}{m-1}\sum_{l=1}^{m}\left(\lambda_l(T) - \bar{\lambda}(T)\right)^2$$

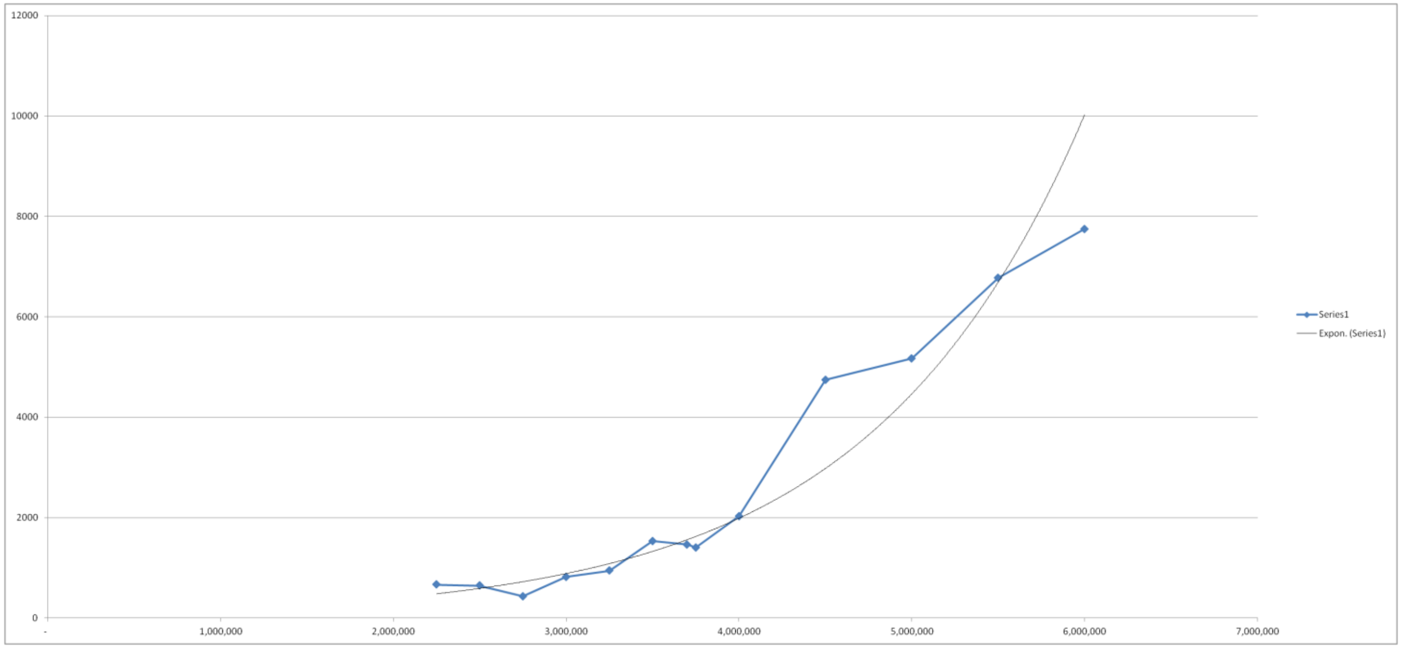
$$=> b(T) = \frac{\bar{\lambda}(T)}{\frac{1}{m-1}\sum_{l=1}^{m}\left(\lambda_l(T) - \bar{\lambda}(T)\right)^2}$$

# *Estimation of the parameters-Frequency of cedents*



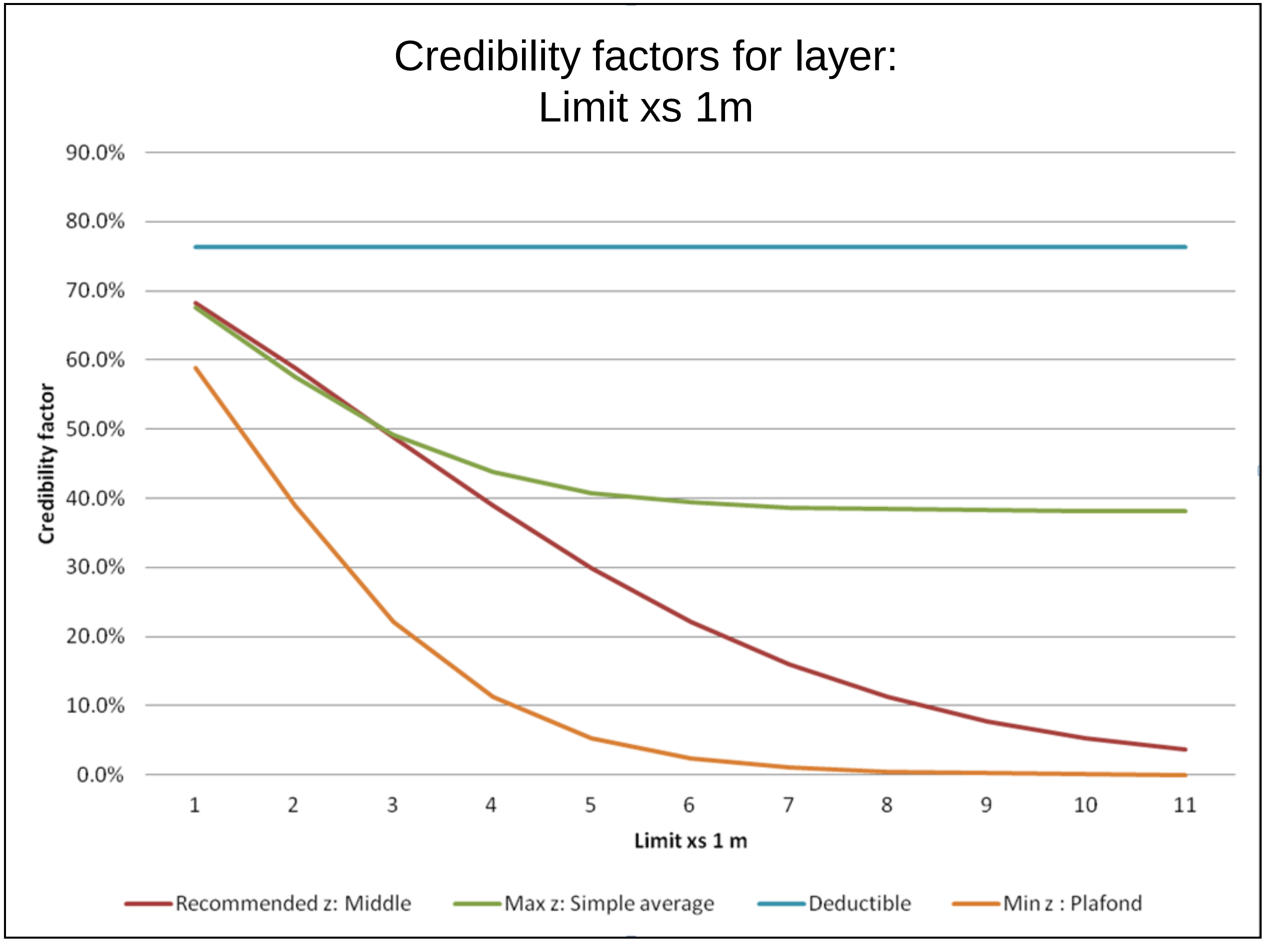

Expected Frequency of cedents @ different thresholds

# *Estimation of the parameters- Fit b(T)*




Estimation of b for different thresholds incl. exponential fit

# *Application on client example*



# *Status quo*

- Where are we?
  - Credibility weight is calculated dependent on claim size and exposure
  - Calibrated on frequencies
  - Applied to the rate
  - Underwriting jugdement is possible, because of the range given for the weight
  - Uncertainty of rate not explicitely taken into account, but within underwriting judgement

# *Vision*

# *Vision*

- Where to go?
  - Application for severity
  - Incorporation of market rate uncertainty
  - Expand application towards loadings (capital intensities)
- Other approaches in actuarial literature:
  - application on loss development factors (Pinot/Gogol)
  - making use of lower layers for upper layers

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# *Thank you for your attention*