Reserve Risk Dependencies

under Solvency II and IFRS 4 perspective

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Types of insurance risks

SST and Solvency II

prior year risk (PY-risk) or reserving risk:
The risk of a huge negative claims development result in the next year-end closing. Or with other words, the risk of much too few claim reserves.

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The risk of having much more current year incurred losses than premium.

IFRS 4

risk margin:
Should cover the uncertainty within the (discounted) future cash flows corresponding to insurance liabilities (includes already happened claims as well as future claims of already existing contracts).

Modelling insurance risk

A rough sketch of the classical way

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2. Model each risk for each portfolio.
3. Aggregate the results via a correlation matrix.

Problem: How can we estimate such correlation matrix?

Correlation matrices in use

SST

		PY: MFH	MFK	Sach	ES-Poo	Haft	UVG	UVG Rei	U.o.UVG	KolK	EinK	Trans	Luft	F&K	Rechts	Andere
PY	MFH	1.00	0.15	0.15	0.15	0.25	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	MFK	0.15	1.00	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.15	1.00	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.15	0.15	1.00	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.15	0.15	1.00	0.25	0.00	0.25	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.50	0.15	0.15	0.15	0.25	1.00	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.50	0.15	0.15	0.15	0.25	0.50	0.00	1.00	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	1.00	0.15	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	1.00	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	1.00	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	1.00	0.15	0.15	0.25
	F&K	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	1.00	0.15	0.25
	Rechts	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	1.00	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	1.00
CY	MFH	0.50	0.50	0.15	0.15	0.25	0.25	0.00	0.25	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	MFK	0.50	0.50	0.25	0.25	0.15	0.25	0.00	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.25	0.50	0.25	0.25	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.25	0.25	0.50	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.25	0.15	0.50	0.15	0.00	0.15	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.50	0.25	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.25	0.50	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.50	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	0.50	0.15	0.15	0.25
	F&K	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.50	0.15	0.25
	Rechts	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	0.50	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.50

		CY: MFH	MFK	Sach	ES-Pool	Haft	UVG	UVG Ren	U.o.UVG	KolK	EinK	Trans	Luft	F&K	Rechts	Andere
PY	MFH	0.50	0.50	0.15	0.15	0.25	0.25	0.00	0.25	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	MFK	0.50	0.50	0.25	0.25	0.15	0.25	0.00	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.25	0.50	0.25	0.25	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.25	0.25	0.50	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.25	0.15	0.50	0.15	0.00	0.15	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.50	0.25	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.25	0.50	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.50	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	0.50	0.15	0.15	0.25
	F&K	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.50	0.15	0.25
	Rechts	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	0.50	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.50
CY	MFH	1.00	0.50	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	MFK	0.50	1.00	0.25	0.25	0.15	0.25	0.00	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.25	1.00	0.25	0.25	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.25	0.25	1.00	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.25	0.15	1.00	0.15	0.00	0.15	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.25	0.25	0.15	0.15	0.15	1.00	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	1.00	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.15	0.15	0.15	0.15	0.15	0.25	0.00	0.25	1.00	0.25	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.25	1.00	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	1.00	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	1.00	0.15	0.15	0.25
	F&K	0.15	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	1.00	0.15	0.25
	Rechts	0.15	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	1.00	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	1.00

100% 50% 25% 15% 0%

Correlation matrices in use

SST

		PY														
		MFH	MFK	Sach	ES-Poo	Haft	UVG	UVG Rei	U.o.UVG	KolK	EinK	Trans	Luft	F&K	Rechts	Andere
PY	MFH	1.00	0.15	0.15	0.15	0.25	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	MFK	0.15	1.00	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.15	1.00	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.15	0.15	1.00	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.15	0.15	1.00	0.25	0.00	0.25	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.50	0.15	0.15	0.15	0.25	1.00	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.50	0.15	0.15	0.15	0.25	0.50	0.00	1.00	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	1.00	0.15	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	1.00	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	1.00	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	1.00	0.15	0.15	0.25
	F&K	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	1.00	0.15	0.25
	Rechts	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	1.00	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	1.00
CY	MFH	0.50	0.50	0.15	0.15	0.25	0.25	0.00	0.25	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	MFK	0.50	0.50	0.25	0.25	0.15	0.25	0.00	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.25	0.50	0.25	0.25	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.25	0.25	0.50	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.25	0.15	0.50	0.15	0.00	0.15	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.50	0.25	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.25	0.50	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.50	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	0.50	0.15	0.15	0.25
	F&K	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.50	0.15	0.25
	Rechts	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	0.50	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.50

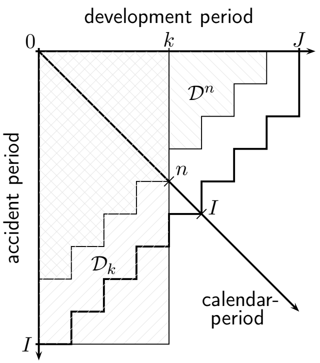
		CY														
		MFH	MFK	Sach	ES-Pool	Haft	UVG	UVG Ren	U.o.UVG	KolK	EinK	Trans	Luft	F&K	Rechts	Andere
PY	MFH	0.50	0.50	0.15	0.15	0.25	0.25	0.00	0.25	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	MFK	0.50	0.50	0.25	0.25	0.15	0.25	0.00	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.25	0.50	0.25	0.25	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.25	0.25	0.50	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.25	0.15	0.50	0.15	0.00	0.15	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.50	0.25	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.25	0.50	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	0.50	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	0.50	0.15	0.15	0.25
	F&K	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.50	0.15	0.25
	Rechts	0.25	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	0.50	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.50
CY	MFH	1.00	0.50	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	MFK	0.50	1.00	0.25	0.25	0.15	0.25	0.00	0.25	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Sach	0.15	0.25	1.00	0.25	0.25	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	ES-Pool	0.15	0.25	0.25	1.00	0.15	0.15	0.00	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.25
	Haft	0.25	0.15	0.25	0.15	1.00	0.15	0.00	0.15	0.15	0.15	0.15	0.25	0.15	0.15	0.25
	UVG	0.25	0.25	0.15	0.15	0.15	1.00	0.00	0.50	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	UVG Renten	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	U.o.UVG	0.25	0.25	0.15	0.15	0.15	0.50	0.00	1.00	0.25	0.15	0.15	0.25	0.25	0.25	0.25
	KolK	0.15	0.15	0.15	0.15	0.15	0.25	0.00	0.25	1.00	0.25	0.15	0.15	0.25	0.25	0.25
	EinK	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.25	1.00	0.15	0.15	0.15	0.15	0.25
	Trans	0.15	0.15	0.15	0.15	0.15	0.15	0.00	0.15	0.15	0.15	1.00	0.15	0.15	0.15	0.25
	Luft	0.25	0.15	0.15	0.15	0.25	0.25	0.00	0.25	0.15	0.15	0.15	1.00	0.15	0.15	0.25
	F&K	0.15	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	1.00	0.15	0.25
	Rechts	0.15	0.15	0.15	0.15	0.15	0.25	0.00	0.25	0.25	0.15	0.15	0.15	0.15	1.00	0.25
	Andere	0.25	0.25	0.25	0.25	0.25	0.25	0.00	0.25	0.25	0.25	0.25	0.25	0.25	0.25	1.00

100% 50% 25% 15% 0%

Solvency II (QIS 5)

	1	2	3	4	5	6	7	8	9	10	11	12
1: Motor vehicle liability	1.00	0.50	0.50	0.25	0.50	0.25	0.50	0.25	0.50	0.25	0.25	0.25
2: Other motor	0.50	1.00	0.25	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.25	0.25
3: MAT	0.50	0.25	1.00	0.25	0.25	0.25	0.25	0.50	0.50	0.25	0.25	0.50
4: Fire	0.25	0.25	0.25	1.00	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.50
5: 3rd party liability	0.50	0.25	0.25	0.25	1.00	0.50	0.50	0.25	0.50	0.25	0.50	0.25
6: Credit	0.25	0.25	0.25	0.25	0.50	1.00	0.50	0.25	0.50	0.25	0.50	0.25
7: Legal exp.	0.50	0.50	0.25	0.25	0.50	0.50	1.00	0.25	0.50	0.25	0.50	0.25
8: Assistance	0.25	0.50	0.50	0.50	0.25	0.25	0.25	1.00	0.50	0.50	0.25	0.25
9: Miscellaneous.	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	1.00	0.25	0.25	0.50
10:Np reins. (property)	0.25	0.25	0.25	0.50	0.25	0.25	0.25	0.50	0.25	1.00	0.25	0.25
11:Np reins. (casualty)	0.25	0.25	0.25	0.25	0.50	0.50	0.50	0.25	0.25	0.25	1.00	0.25
12:Np reins. (MAT)	0.25	0.25	0.50	0.50	0.25	0.25	0.25	0.25	0.50	0.25	0.25	1.00

Definition 2.1 (σ-algebras)



- $\mathcal{B}_{i,k}$ is the σ-algebra of all information of accident period i up to development period k:
$$\mathcal{B}_{i,k} := \sigma\left(S_{i,j} \colon 0 \le j \le k\right) = \sigma\left(C_{i,j} \colon 0 \le j \le k\right)$$
- $\mathcal{D}^n$ is the σ-algebra of all information up to calender period n:
$$\begin{aligned}\mathcal{D}^n :=& \sigma\left(S_{i,k} \colon 0 \le i \le I,\, 0 \le k \le J \wedge (n-i)\right)\\ =& \sigma\left(C_{i,k} \colon 0 \le i \le I,\, 0 \le k \le J \wedge (n-i)\right)\\ =& \sigma\left(\bigcup_{i=0}^{I} \bigcup_{k=0}^{J\wedge(n-i)} \mathcal{B}_{i,k}\right)\end{aligned}$$
- $\mathcal{D}_k$ is the σ-algebra of all information up to development period k:
$$\begin{aligned}\mathcal{D}_k :=& \sigma\left(S_{i,j} \colon 0 \le i \le I,\, 0 \le j \le k\right)\\ =& \sigma\left(C_{i,j} \colon 0 \le i \le I,\, 0 \le j \le k\right)\\ =& \sigma\left(\bigcup_{i=0}^{I} \mathcal{B}_{i,k}\right)\end{aligned}$$
- $\mathcal{D}^n_k := \sigma\left(\mathcal{D}^n \cup \mathcal{D}_k\right)$

Chain-Ladder-Method (CLM)

Actuaries often use the Chain-Ladder-Method for reserving. That means they believe in

i)$^{\text{CLM}}$ $\mathsf{E}\left[C_{i,k+1}|\mathcal{D}_k^{i+k}\right] = f_k C_{i,k}$,

ii)$^{\text{CLM}}$ $\mathsf{Var}\left[C_{i,k+1}|\mathcal{D}_k^{i+k}\right] = \sigma_k^2 C_{i,k}$ and

iii)$^{\text{CLM}}$ accident periods are independent.

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Idea

Why not take several triangles $C_{i,k}^m$, $0 \leq m \leq M$ and couple them via

$$\mathrm{Cov}\left[C_{i,k+1}^{m_1}, C_{i,k+1}^{m_2}\middle|\mathcal{D}_k^{i+k}\right] = \sigma_k^{m_1,m_2}\sqrt{C_{i,k}^{m_1} C_{i,k}^{m_2}}\ ?$$

Extended-Complementary-Loss-Ratio-Method (ECLRM)

We take incremental payments $S^0_{i,k}$, changes in reported amounts $S^1_{i,k}$, case reserves $R_{i,k}$ and assume that

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Why not take several portfolios and couple them via

$$\mathrm{Cov}\left[S^{m_1}_{i,k+1}, S^{m_2}_{i,k+1}\middle|\mathcal{D}^{i+k}_k\right] = \sigma^{m_1,m_2}_k \sqrt{R^{m_1}_{i,k} R^{m_2}_{i,k}} \;?$$

Other examples

Similar statements can be formulated for

- the Bornhuetter-Ferguson-Method,
- the Complementary-Loss-Ratio-Method,
- the Cape-Cod-Method,
- the Benktander-Hovinen-Method
- ...

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- ...

What do have those methods in common?

- the expectation of next years development is proportional to some exposure, which is a linear combination of the past developments,
- covariances are proportional to some exposure, which depends only on the past developments.

Linear-Stochastic-Reserving-Methods (LSRMs)

We have several claim properties (triangles) $S_{i,k}^{m}$ and assume that:

i)$^{\text{LSRM}}$ There exist exposures $R_{i,k}^{m} \in \mathcal{D}^{i+k} \cap \mathcal{D}_k$, which depend linearly on claim properties $\mathbf{S}$, such that

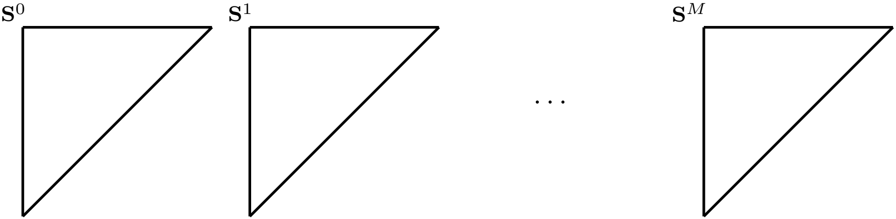
$$\mathsf{E}\left[S_{i,k+1}^{m}\middle|\mathcal{D}_k^{i+k}\right] = f_k^m R_{i,k}^m := f_k^m \Gamma_{i,k}^m \mathbf{S}.$$

ii)$^{\text{LSRM}}$ There exist exposures $R_{i,k}^{m_1,m_2} \in \mathcal{D}^{i+k} \cap \mathcal{D}_k$ such that

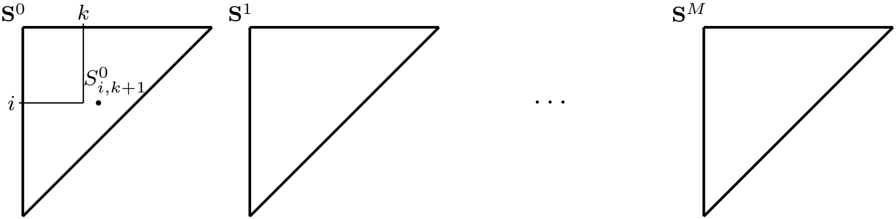
$$\mathsf{Cov}\left[S_{i,k+1}^{m_1}, S_{i,k+1}^{m_2}\middle|\mathcal{D}_k^{i+k}\right] = \sigma_k^{m_1,m_2} R_{i,k}^{m_1,m_2}.$$

An updated version of the original paper, see [2], can be obtained from the author.

LSRM step by step



LSRM step by step



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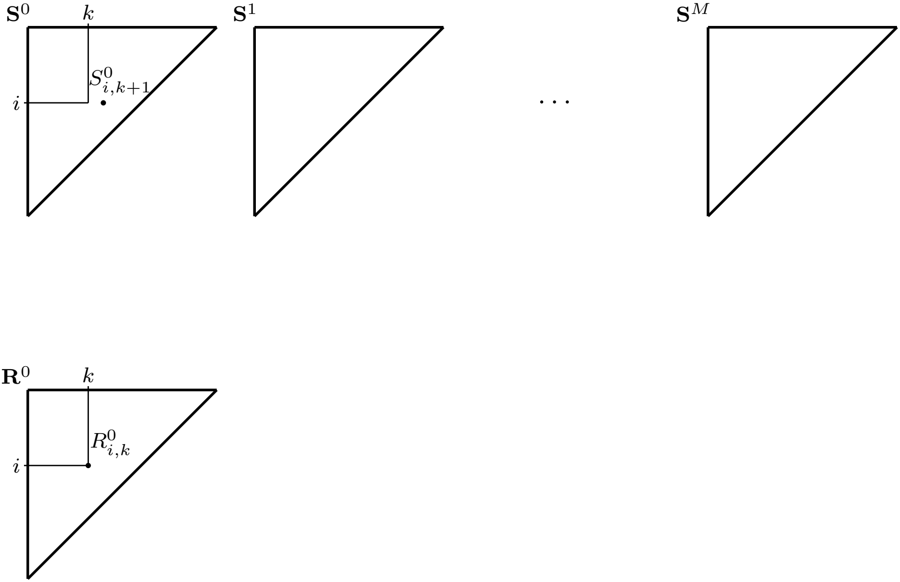

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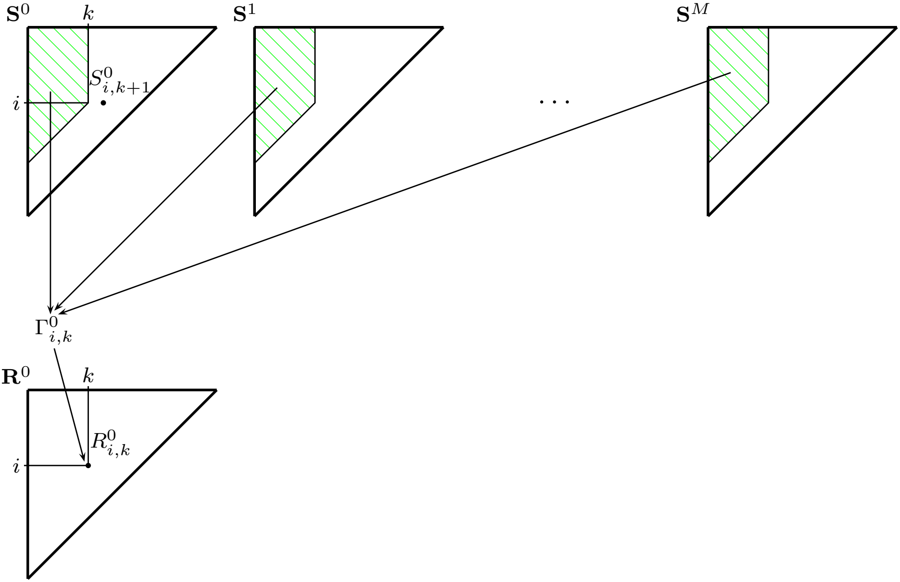

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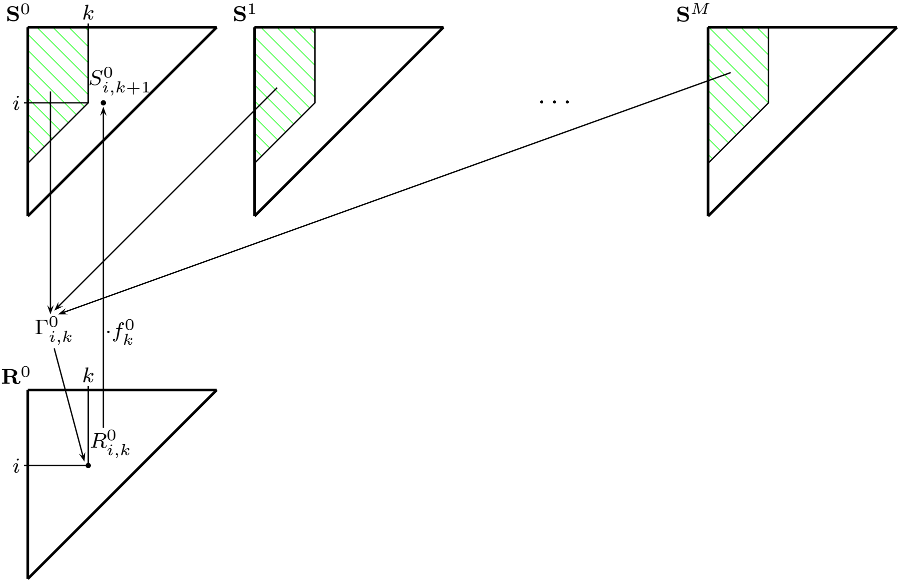

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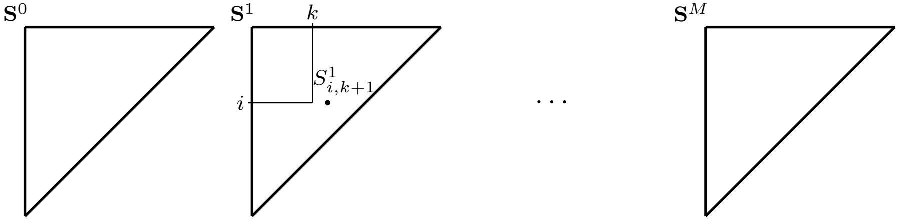
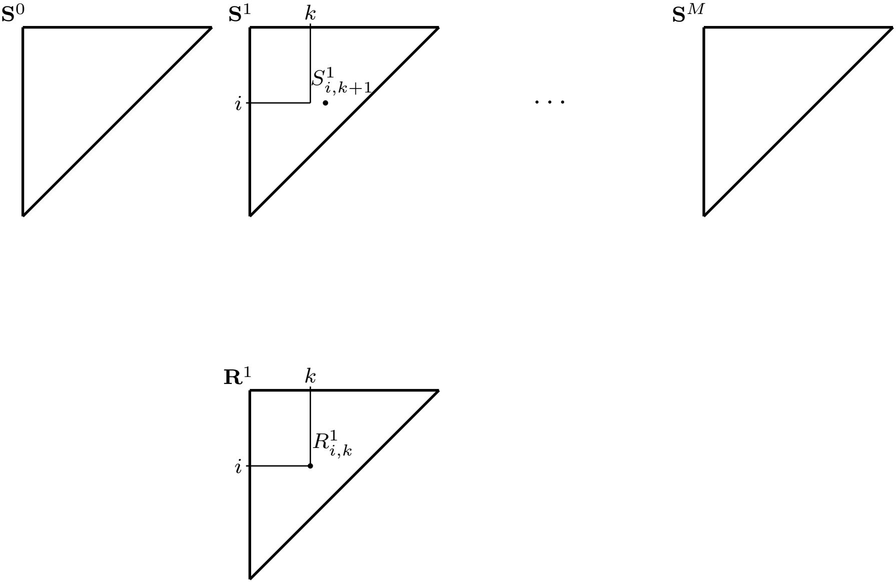


LSRM step by step



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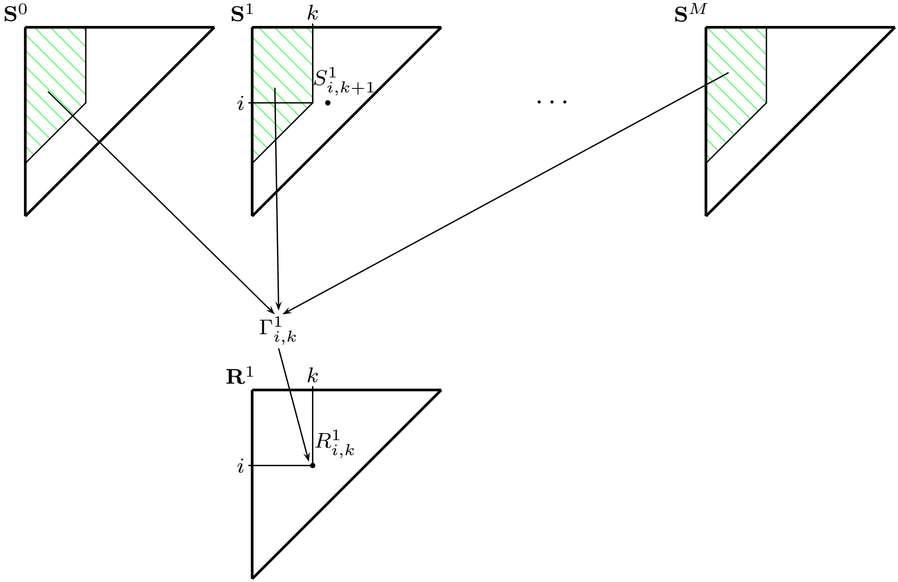

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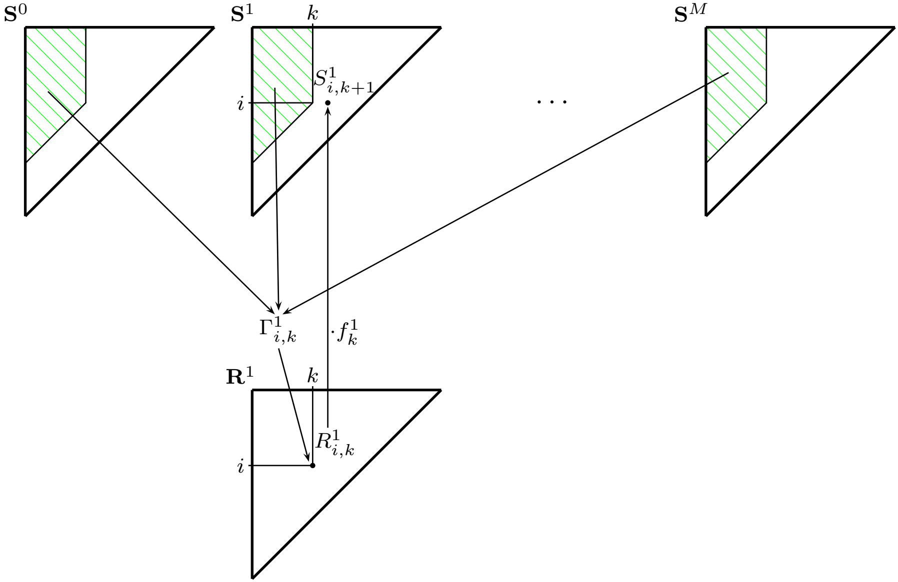

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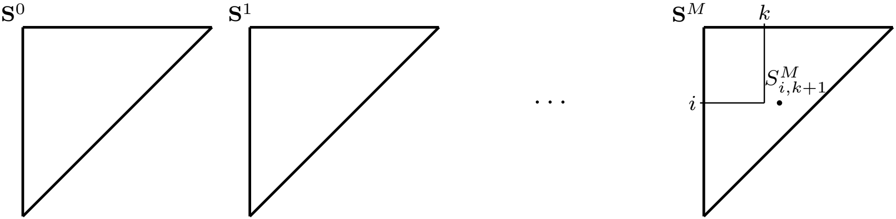

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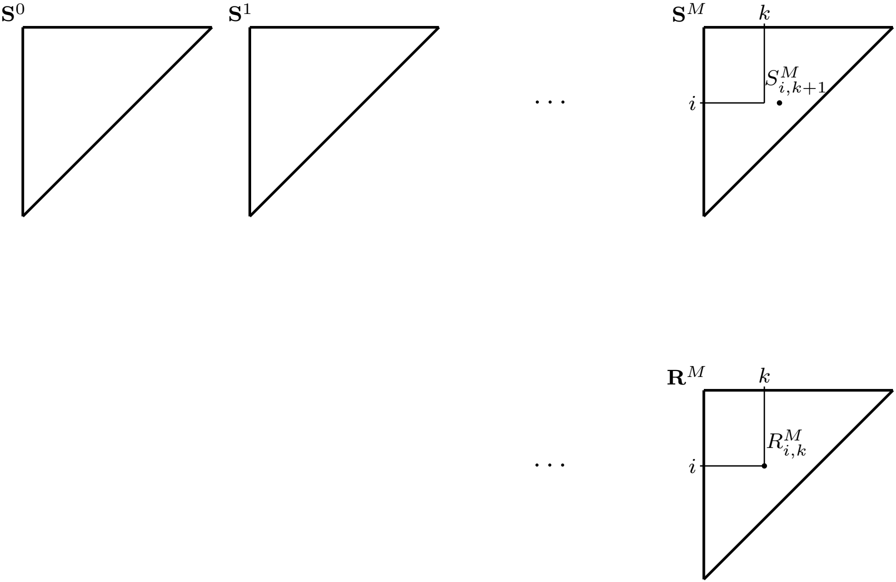

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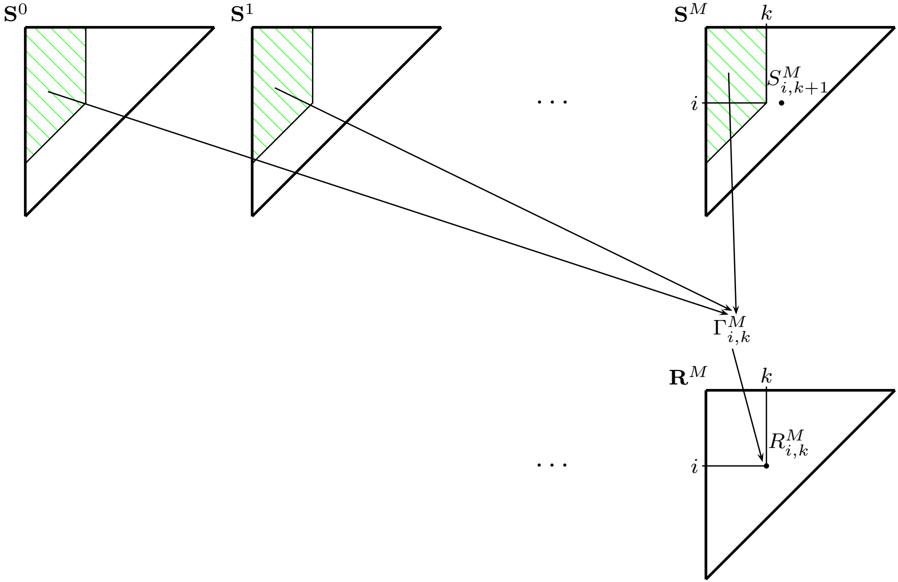
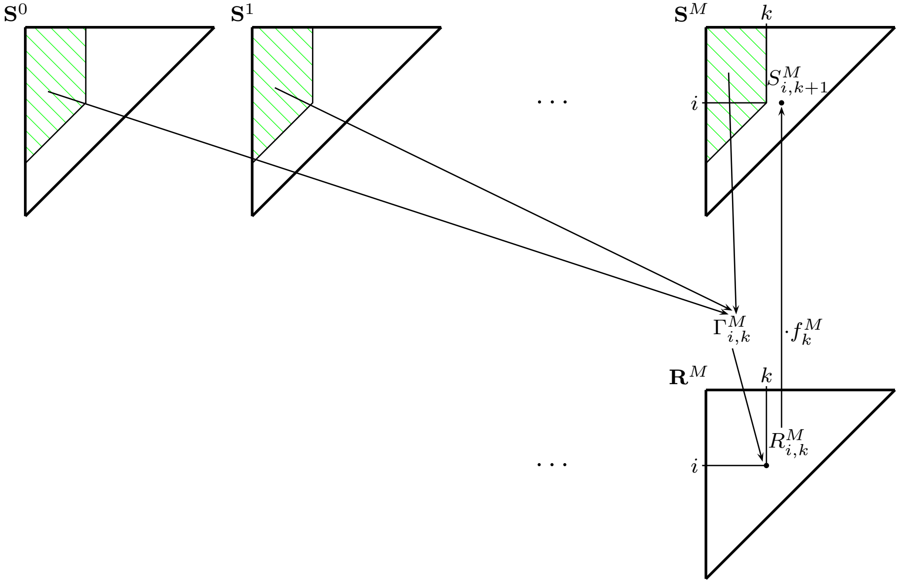


LSRM step by step



Modelling all portfolios at once

We can look at the total ultimate

$$\sum_{m=0}^{M}\sum_{i=0}^{I}\alpha_i^m\sum_{k=0}^{J}S_{i,k}^m,$$

where α_i^m are arbitrary real numbers (mixing weights).
Then the variance of the reserving risk can be estimated by the mean squared error of prediction (mse), which is of the form

$$\widehat{\mathsf{mse}} = \sum_{m_1,m_2=0}^{M}\sum_{i_1,i_2=0}^{I}\alpha_{i_1}^{m_1}\alpha_{i_2}^{m_2}\widehat{\beta}_{i_1,i_2}^{m_1,m_2}.$$

That is true for the ultimate reserving risk as well as for the solvency reserving risk (with different β's of course).

Reverse engineering of a correlation matrix

If we are required to use a correlation approach we could use the components of overall $\widehat{\mathrm{mse}}$ in order to define the correlation matrix, i.e. we could take

$$\left(\frac{\sum_{i_1,i_2=0}^{I} \alpha_{i_1}^{m_1} \alpha_{i_2}^{m_2} \widehat{\beta}_{i_1,i_2}^{m_1,m_2}}{\sqrt{\sum_{i=0}^{I} \alpha_i^{m_1} \alpha_i^{m_1} \widehat{\beta}_{i,i}^{m_1,m_1} \sum_{i=0}^{I} \alpha_i^{m_2} \alpha_i^{m_2} \widehat{\beta}_{i,i}^{m_2,m_2}}} \right)^{0 \le m_1, m_2 \le M}$$

as correlation matrix.

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as correlation matrix.

Model error

Since the real world does not entirely follow the assumption on LSRMs, our estimations of the reserve risk should be increased by an model error.

Fire non commercial vs. motor own damage

correlation	ultimate	solvency
SST		15 %
QIS 5		25 %
mixed CLM	20 %	25 %
CLM on paid	25 %	35 %
CLM on incurred	20 %	30 %
ECLRM	-5 %	-5 %

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Motor TPL vs. motor own damage

correlation	ultimate	solvency
SST		15 %
QIS 5		50 %
mixed CLM	5 %	5 %
CLM on paid	0 %	0 %
CLM on incurred	10 %	15 %
ECLRM	10 %	20 %

Including CY-risk

This month a master student will start to investigate the possibilities to include the CY-risk. The basic idea is:

- add a column at the left side of each triangle that corresponds to the estimated ultimate of the next period (CY ultimate).
- explore different exposures with respect to stability and comparability.

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LSRM-Tools

There is a runtime library that implements LSRMs under the public licence GPL 3. Moreover, it includes an Excel-Add-In that allows easy access to these futures. The examples of this presentation have been generated by using these tools. All can be obtained from the author.

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